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STOCHASTIC ORDERING

IN RESIDUAL MIXING DISTRIBUTIONS

by

David R. Campbell

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A mixture of failure rates can be present in an apparently homogeneous population of "devices" because of variability either in their manufacture or in the severity of their service environments. A mixing distribution is the probability distribution for different failure rates in such a population. A residual mixing distribution is the probability distribution for different failure rates in the population of surviving devices after a specified period of service or "burn in."

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Residual mixing distributions resulting from arbitrary mixtures of constant failure rates are shown to be stochastically ordered (decreasingly) as the period of service or burn in is increased, and to approach in the limit a distribution degenerate at the smallest failure rate "present" in the population. The results are direct applications of basic propositions concerning monotonicity properties of residual mixing random variables and their expectations. They contain some well known results about the failure rate of an item drawn from a mixed population of constant failure rates as immediate consequences.

1. Introduction.

Motivated by applications to reliability theory, the model we shall consider is a mixture of the form

$$\bar{F}_T(t) = \int_0^\infty e^{-\lambda t} dG(\lambda), \qquad t \geq 0 \tag{1.1}$$

where $\bar{F}_T(t)$ is the survival function of the mixture, $\bar{F}(t;\lambda) = e^{-\lambda t}$ is the survival function of the component distribution given that $\Lambda = \lambda$ for Λ a nonnegative random variable having cdf (cumulative distribution function) G. We can refer to such a mixture as a G-mixture of exponentials. In the reliability context, T represents the time to failure of an item drawn at random from a population of items with different exponential life distributions, mixed according to the distribution of Λ.

We take the usual definition for a failure rate function, i.e. for the random variable T having density $f_T(t)$ and survival function $\bar{F}_T(t)$, the failure rate function is defined by

$$h_T(t) = \frac{f_T(t)}{\bar{F}_T(t)}, \quad \text{for all } t \text{ such that } \bar{F}_T(t) > 0.$$

The initial mixing distribution G may be modified to yield a related distribution at any time t, i.e. let $G_t(\lambda) = P(\Lambda \leq \lambda \mid T > t)$ and define $\Lambda(t)$ to be a random variable having cdf G_t. G_t is the <u>residual mixing distribution</u> at time t. It represents the revised distribution for the failure rate of a randomly drawn item from the mixture, given that the item is still alive at time t. The relationship between G and G_t may be exhibited through a straightforward application of Bayes' Theorem which yields

$$dG_t(\lambda) = \frac{e^{-\lambda t}}{E(e^{-\Lambda t})} \, dG(\lambda), \tag{1.2}$$

where the expectation is with respect to G, the distribution of Λ.

For such a mixture of exponentials as (1.1), it is well known (Barlow, Marshall, and Proschan [2]) that the mixture failure rate $h_T(t)$ is decreasing in (we shall use 'decreasing' for non-increasing and 'increasing' for non-decreasing), and that (Gnedenko, Belyayev, and Solovyev [3]) the mixture failure rate function approaches as a limit with increasing time t the least of all parameter values positively present in the mixture, i.e. λ_o.

Aldrich and Morton [1] and O'Bar [4] showed that at time t the failure rate function of a mixture of exponentials is the expected value of the residual mixing distribution corresponding to that time, i.e.

$$h_T(t) = E[\Lambda(t)] \tag{1.3}$$

Thus, the sequence of residual mixing distributions is decreasing in expectation. This fact supports the intuitive notion that with increasing time the shorter-lived members of the mixture are being "weeded out."

The conjecture of a compatible stochastic ordering of the residual mixing distributions arises as a logical extension of the foregoing results. In this paper the existence of such a stochastic ordering is demonstrated. It is also shown that the residual mixing distributions converge in distribution to the distribution degenerate at the least parameter value positively present in the mixture, with an attendant monotone convergence of raw moments. The latter result is one asserted by Aldrich and Morton.

The results are presented as applications of Propositions 1 and 2, to be developed in the next section. These propositions state properties of $E\{u[\Lambda(t)]\}$ for fairly general functions u. To indicate the relevance of these propositions, we first note that an expression of this form is involved in (1.3), taking $u[\Lambda(t)] = \Lambda(t)$. We next define an indicator function

$$I_\lambda(\Lambda) = \begin{cases} 0, & \Lambda \leqq \lambda \\ 1, & \Lambda > \lambda \end{cases} \tag{1.4}$$

Using this indicator function, $\bar{G}_t$ may be expressed as

$$\bar{G}_t(\lambda) = \int_\lambda^\infty dG_t(x) = E\{I_\lambda[\Lambda(t)]\}, \tag{1.5}$$

another expression of the same form, with $u[\Lambda(t)] = I_\lambda[\Lambda(t)]$.

2. Basic Results.

We shall develop in Propositions 1 and 2 of this section basic tools which should prove to be useful in a wide variety of applications. Several of these applications are given in section 3.

Using (1.2) we find that

$$\begin{aligned} E\{u[\Lambda(t)]\} &= \int_0^\infty u(\lambda)dG_t(\lambda) \\ &= \int_0^\infty \frac{u(\lambda)e^{-\lambda t}}{E(e^{-\Lambda t})} dG(\lambda) \\ &= \frac{E[u(\Lambda)e^{-\Lambda t}]}{E(e^{-\Lambda t})} . \end{aligned} \tag{2.1}$$

Thus, expectations of functions of $\Lambda(t)$ may be evaluated by simply taking expectations of functions of Λ. A straightforward application of the Monotone Convergence Theorem yields

$$\frac{d}{dt}\{E[u(\Lambda)e^{-\Lambda t}]\} = E[u(\Lambda)\frac{d}{dt}e^{-\Lambda t}]$$

$$= -E[\Lambda u(\Lambda)e^{-\Lambda t}], \qquad (2.2)$$

for $\Lambda \geq 0$.

We next require the function u to be nonnegative and monotone to get

PROPOSITION 1. If Λ is a nonnegative random variable and u a nonnegative monotone increasing (decreasing) function, then

$$\varphi_u(t) = \frac{E[u(\Lambda)e^{-\Lambda t}]}{E(e^{-\Lambda t})} = E\{u[\Lambda(t)]\}$$

is monotone decreasing (increasing) in t.

Proof: Using (2.2),

$$\frac{d}{dt}\varphi_u(t) = \frac{E(e^{-\Lambda t})E[-\Lambda u(\Lambda)e^{-\Lambda t}] - E[u(\Lambda)e^{-\Lambda t}]E[-\Lambda e^{-\Lambda t}]}{E(e^{-\Lambda t})^2}$$

$$= -\frac{E[\Lambda u(\Lambda)e^{-\Lambda t}]}{E(e^{-\Lambda t})} + \frac{E[u(\Lambda)e^{-\Lambda T}]}{E(e^{-\Lambda t})}\cdot\frac{E(\Lambda e^{-\Lambda t})}{E(e^{-\Lambda t})}$$

$$= E\{u[\Lambda(t)]\}E[\Lambda(t)] - E\{\Lambda(t)u[\Lambda(t)]\}$$

$$= -\,\mathrm{COV}\{u[\Lambda(t)],\ \Lambda(t)\}.$$

Then by the proporty of the covariance of similarly/dissimilarly ordered functions of the same random variable,

(i) u increasing in $\Lambda(t) \Rightarrow \frac{d}{dt}\varphi_u(t) \leq 0$, and

(ii) u decreasing in $\Lambda(t) \Rightarrow \frac{d}{dt}\varphi_u(t) \geq 0$. □

We now need two lemmas regarding limits of expectations involving time t as a parameter.

LEMMA 1. Given u a nonnegative function,

$$\lim_{t\to\infty} E[I_\lambda(\Lambda)u(\Lambda)e^{-(\Lambda-\lambda)t}] = 0.$$

Proof: Note that $I_\lambda(\Lambda) = 1 \Leftrightarrow \Lambda - \lambda > 0 \Leftrightarrow e^{-(\Lambda-\lambda)t}\downarrow$ in t. An application of the Monotone Convergence Theorem yields

$$\lim_{t\to\infty} E[I_\lambda(\Lambda)u(\Lambda)e^{-(\Lambda-\lambda)t}] = E[I_\lambda(\Lambda)u(\Lambda)\lim_{t\to\infty} e^{-(\Lambda-\lambda)t}] = 0. \qquad □$$

LEMMA 2. For $\lambda > \lambda_o = \inf\{\lambda \mid \bar{G}(\lambda)<1\}$,

$$\lim_{t\to\infty} E[(1-I_\lambda(\Lambda))e^{-(\Lambda-\lambda)t}] > 0.$$

Proof: Note that $(1-I_\lambda(\Lambda)) = 1 \Leftrightarrow \Lambda - \lambda \leq 0 \Leftrightarrow e^{-(\Lambda-\lambda)t} \geq 1$ for $t \geq 0$. Then $E[(1-I_\lambda(\Lambda))e^{-(\Lambda-\lambda)t}] \geq E[1-I_\lambda(\Lambda)] = G(\lambda)$, and $\lim_{t\to\infty} E[(1-I_\lambda(\Lambda))e^{-(\Lambda-\lambda)t}] \geq G(\lambda) > 0$.

□

If, in addition to the monotonicity requirement of Proposition 1, we also require u to be right-continuous at λ_o, we may find the limiting value of $\varphi_u(t)$ as $t \to \infty$. We state this result in

PROPOSITION 2. Let Λ be a nonnegative random variable having cdf G, and u a nonnegative monotone increasing (decreasing) function. If u is right-continuous at $\lambda_o = \inf\{\lambda \mid \bar{G}(\lambda)<1\}$, then $\varphi_u(t)$ decreases (increases) monotonically to $u(\lambda_o)$ as $t \to \infty$.

<u>Proof</u>:

(i) If $u(\Lambda)\uparrow$ in Λ then $\varphi_u(t) \geq u(\lambda_o)$. By Proposition 1, $\varphi_u(t)\downarrow$ in t. Thus, $\lim_{t\to\infty} \varphi_u(t)$ exists and lies in the interval $[u(\lambda_o),\infty)$. Choose $\lambda > \lambda_o$. Then,

$$\varphi_u(t) = \frac{E[(1-I_\lambda(\Lambda))u(\Lambda)e^{-\Lambda t}] + E[I_\lambda(\Lambda)u(\Lambda)e^{-\Lambda t}]}{E[(1-I_\lambda(\Lambda))e^{-\Lambda t}] + E[I_\lambda(\Lambda)e^{-\Lambda t}]}$$

$$= \frac{E[(1-I_\lambda(\Lambda))u(\Lambda)e^{-(\Lambda-\lambda)t}] + E[I_\lambda(\Lambda)u(\Lambda)e^{-(\Lambda-\lambda)t}]}{E[(1-I_\lambda(\Lambda))e^{-(\Lambda-\lambda)t}] + E[I_\lambda(\Lambda)e^{-(\Lambda-\lambda)t}]},$$

Since $u(\Lambda)\uparrow$ in Λ,

$$\varphi_u(t) \leq \frac{u(\lambda)E[(1-I_\lambda(\Lambda))e^{-(\Lambda-\lambda)t}] + E[I_\lambda(\Lambda)u(\Lambda)e^{-(\Lambda-\lambda)t}]}{E[(1-I_\lambda(\Lambda))e^{-(\Lambda-\lambda)t}] + E[I_\lambda(\Lambda)e^{-(\Lambda-\lambda)t}]},$$

$$\leq u(\lambda) + \frac{E[I_\lambda(\Lambda)u(\Lambda)e^{-(\Lambda-\lambda)t}]}{E[(1-I_\lambda(\Lambda))e^{-(\Lambda-\lambda)t}] + E[I_\lambda(\Lambda)e^{-(\Lambda-\lambda)t}]}, \tag{1}$$

Lemmas 1 and 2 may be applied to show that the last term in expression (1) goes to zero as $t \to \infty$. Therefore, $\lim_{t\to\infty} \varphi_u(t) \leq u(\lambda)$. Let $\lambda \downarrow \lambda_o$ to get $\lim_{t\to\infty} \varphi_u(t) \leq u(\lambda_o)$. Then, $\varphi_u(t) \downarrow u(\lambda_o)$ as $t \to \infty$.

(ii) The proof for u monotone decreasing is similar to that above. □

3. <u>Applications</u>.

The first application of the tools developed in the preceding section concerns the stochastic ordering of the sequence of residual mixing distributions and its limiting distribution, stated as

<u>Theorem 1</u>. If Λ is a nonnegative random variable with cdf G and $\lambda_o = \inf\{\lambda \mid \bar{G}(\lambda)<1\}$, then for a G-mixture of exponentials

a) G_t is a stochastically decreasing sequence in t, i.e.

$$t_1 \le t_2 \Rightarrow \bar{G}_{t_1}(\lambda) \ge \bar{G}_{t_2}(\lambda), \quad \text{for all } \lambda.$$

b) G_t converges in distribution to the distribution degenerate at λ_o, as $t \to \infty$.

<u>Proof</u>:

a) By (1.5),

$$\bar{G}_t(\lambda) = E\{I_\lambda[\Lambda(t)]\}.$$

Note that I_λ is monotone increasing in its argument. Application of Proposition 1 with $u[\Lambda(t)] = I_\lambda[\Lambda(t)]$ yields $\bar{G}_t(\lambda)\downarrow$ in t.

b) Application of Proposition 2 with $u[\Lambda(t)] = I_\lambda[\Lambda(t)]$ yields as $t \to \infty$,

$$\bar{G}_t(\lambda) \to I_\lambda(\lambda_o) = \begin{cases} 0, & \lambda_o \le \lambda \\ 1, & \lambda_o > \lambda \, . \end{cases} \qquad \square$$

An immediate consequence of Proposition 2 is the well-known result that a mixture of exponentials is DFR, with its failure rate approaching the least parameter value positively present in the mixture. We state this result as

<u>Theorem 2</u>. If Λ is a nonnegative random variable with cdf G and $\lambda_o = \inf\{\lambda \mid \bar{G}(\lambda)<1\}$, then for a G-mixture of exponentials the mixture failure rate function, $h_T(t)$, decreases monotonically to λ_o as $t \to \infty$.

Proof: As noted in (1.3),

$$h_T(t) = E[\Lambda(t)].$$

Application of Proposition 2 with $u[\Lambda(t)] = \Lambda(t)$ yields the desired result.

$\square$

We may also see the monotone convergence for the raw moments of the residual mixing distribution in

Theorem 3. For $p \geq 0$, $E[\Lambda(t)^p]$ decreases monotonically to λ_o^p.

Proof: Application of Proposition 2 with $u[\Lambda(t)] = \Lambda(t)^p$ yields the desired result. $\square$

The variance of $\Lambda(t)$ is of interest, since it has an interpretation as the negative of the slope of the mixture failure rate function (O'Bar [4]). We state the following as

Corollary 1. The limit as $t \to \infty$ of $VAR[\Lambda(t)]$ is zero.

Proof: We express the variance of $\Lambda(t)$ as

$$VAR[\Lambda(t)] = E[\Lambda(t)^2] - E^2[\Lambda(t)] \tag{3.1}$$

Then, by Theorem 3,

$$\lim_{t\to\infty} VAR[\Lambda(t)] = \lambda_o^2 - \lambda_o^2 = 0. \qquad \square$$

Expression (3.1) leads us to

Corollary 2. On the interval $(0,\infty)$, $VAR[\Lambda(t)]$ is of bounded variation.

Proof: From the power series expansion for $e^{\Lambda t}$, we have that

$$\Lambda^n < \frac{n!}{t^n} e^{\Lambda t},$$

for n a nonnegative integer. Thus,

$$E[\Lambda(t)^n] = \frac{E[\Lambda^n e^{-\Lambda t}]}{E[e^{-\Lambda t})} < \frac{n!}{t^n} \frac{E[e^{\Lambda t} e^{-\Lambda t}]}{E(e^{-\Lambda t})}$$

$$< \frac{n!}{t^n E(e^{-\Lambda t})}, \quad \text{for} \quad t > 0.$$

Then, from expression (3.1) and Theorem 3, since $VAR[\Lambda(t)]$ is expressed as the difference of two real-valued nonnegative monotone functions for $t > 0$, it is of bounded variation on $(0,\infty)$. $\square$

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